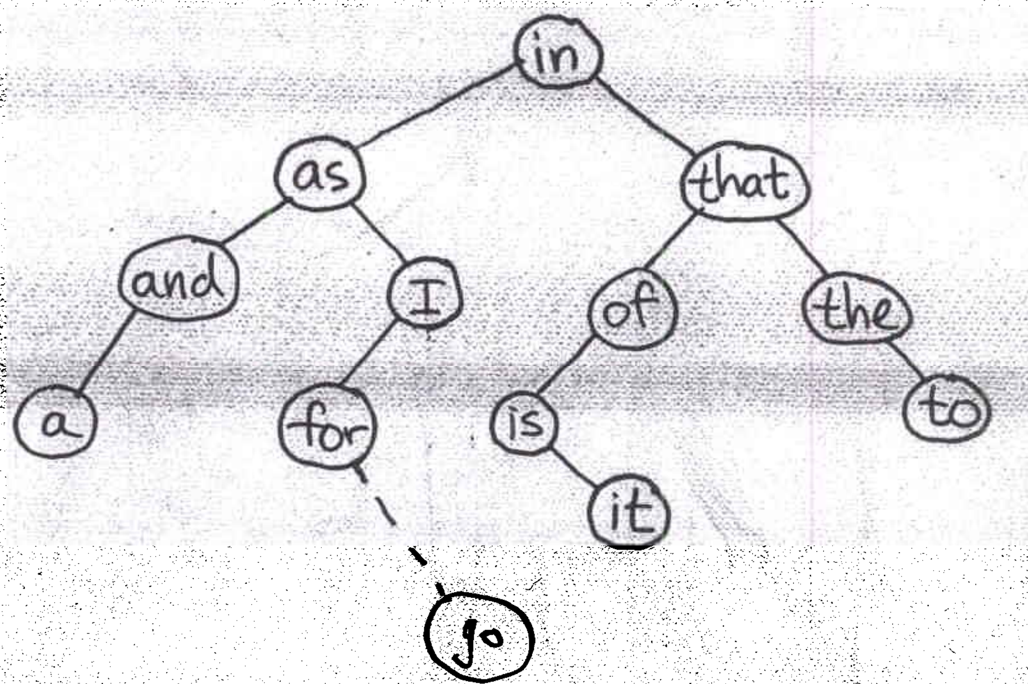
Binary Search Trees

Binary Tree: A rooted tree, each node having a left and a right child, either or both missing.

Binary Search Tree: Each node contains an item. Items are totally ordered and arranged in the tree in symmetric order: all items in left subtree are less, all items in right subtree are greater.

Binary search trees support access, insert, delete in O(depth) time.

A binary seach tree



<u>Classical</u> <u>answer</u>: Maintain a (local) balance condition.

Two properties:

(i) Implies $O(\log n)$ depth of an n-node tree.

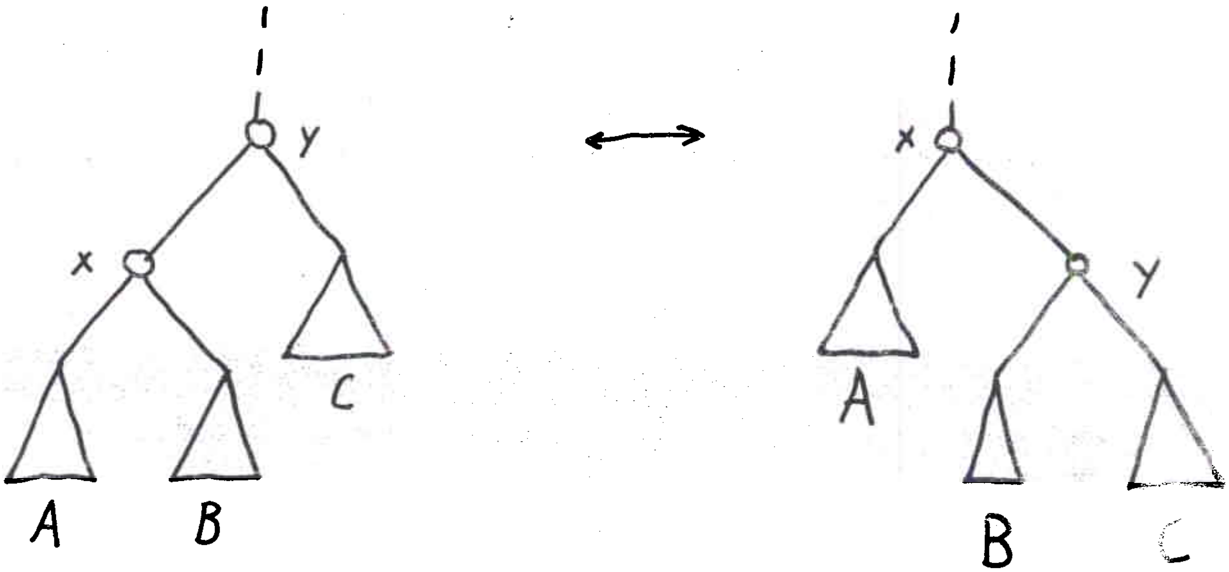
(ii) Easily restorable after an update: $O(\log n)$ time by rebalancing along access path.

Since ~1962 many kinds of such

<u>balanced</u> <u>search</u> <u>trees</u>

have been discovered.

A Rotation

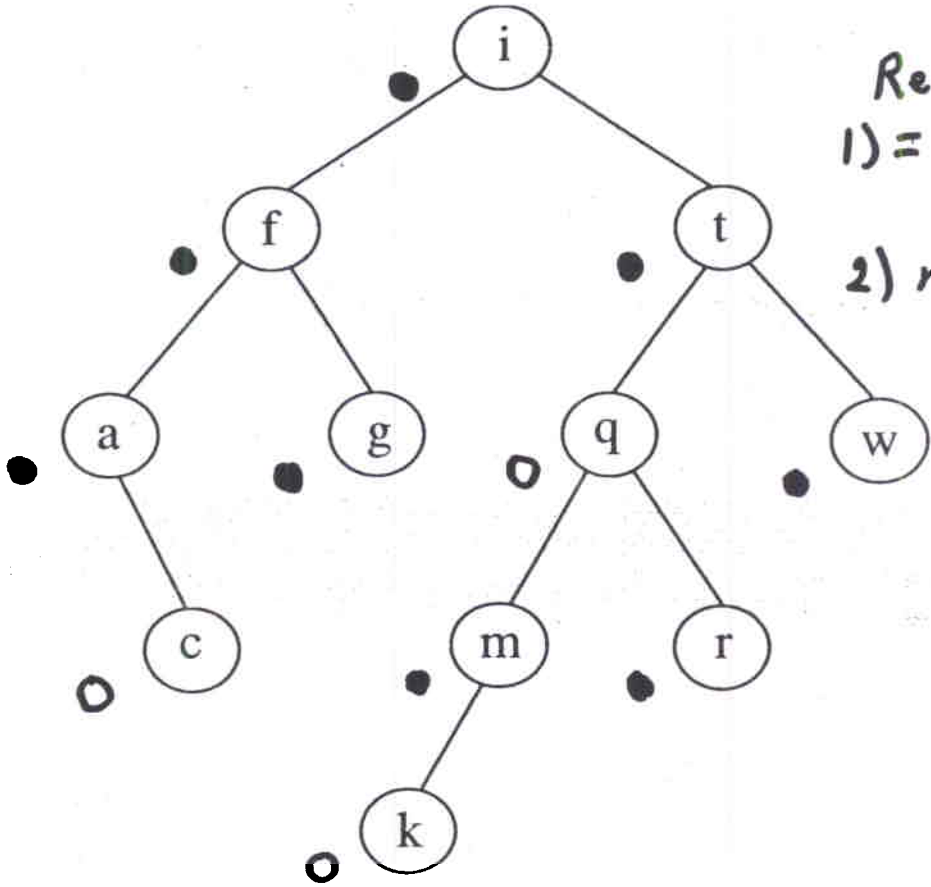


Changes depths of some nodes

Takes $O(1)$ time (3 pointer changes)

Preserves symmetric order

A Binary Search Tree



Red / Black:
1) = #s blacks on paths,
2) red nodes have black parents

Items in internal nodes, in symmetric order:

items in left subtree smaller,

items in right subtree larger.

Allows binary search for items

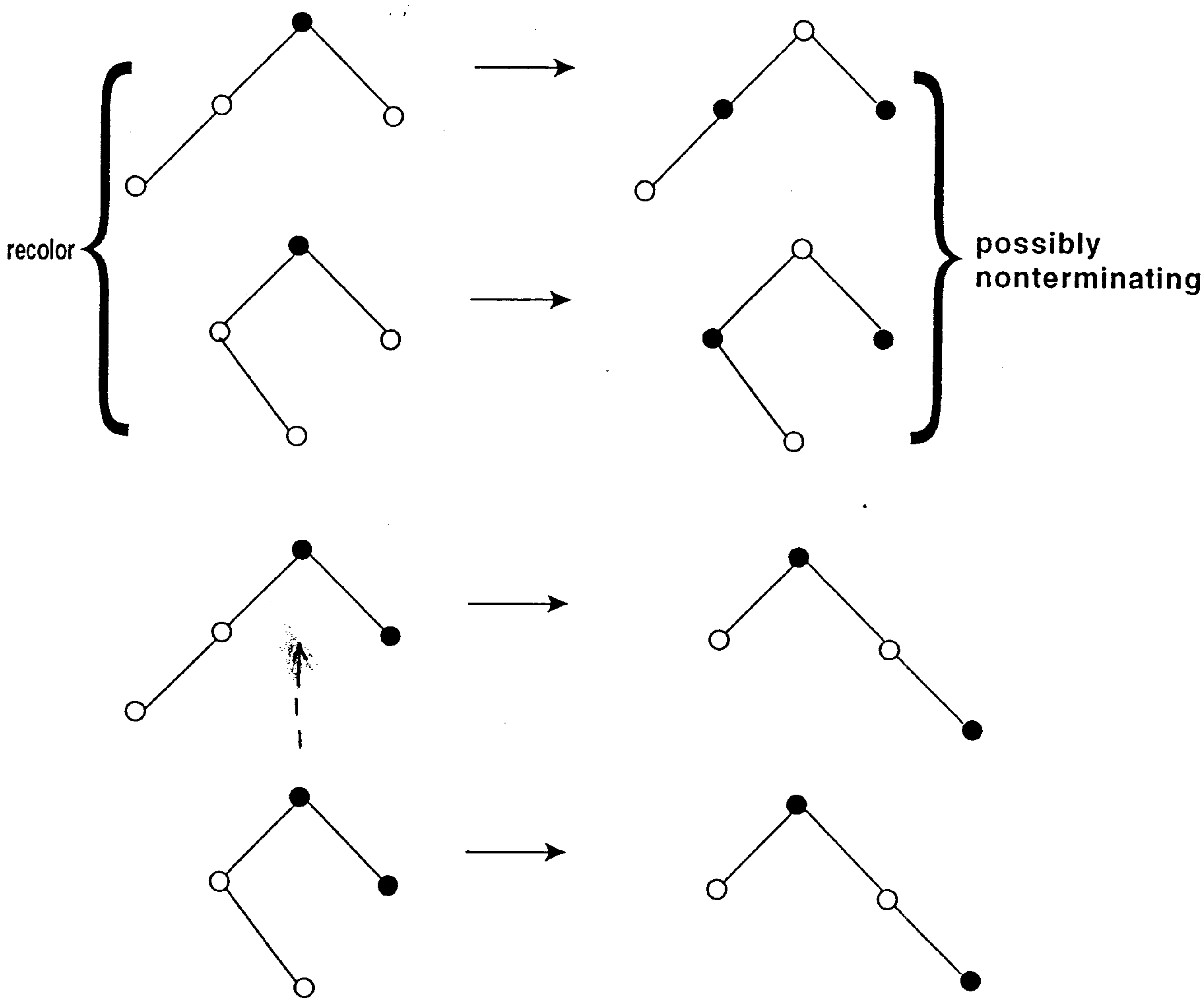
search time = 1+ depth.

Red-black tree updates

● black

○ red

Insert ○ root ⟶ ●



Persistent Search Trees

How do we preserve old versions of tree, allowing queries in the past, updates in the present (and possibly in the past)?

Objective: Avoid copying the entire tree. (takes $O(n)$ space and time per update)

Applications

Text editing*

Applicative programming languages*

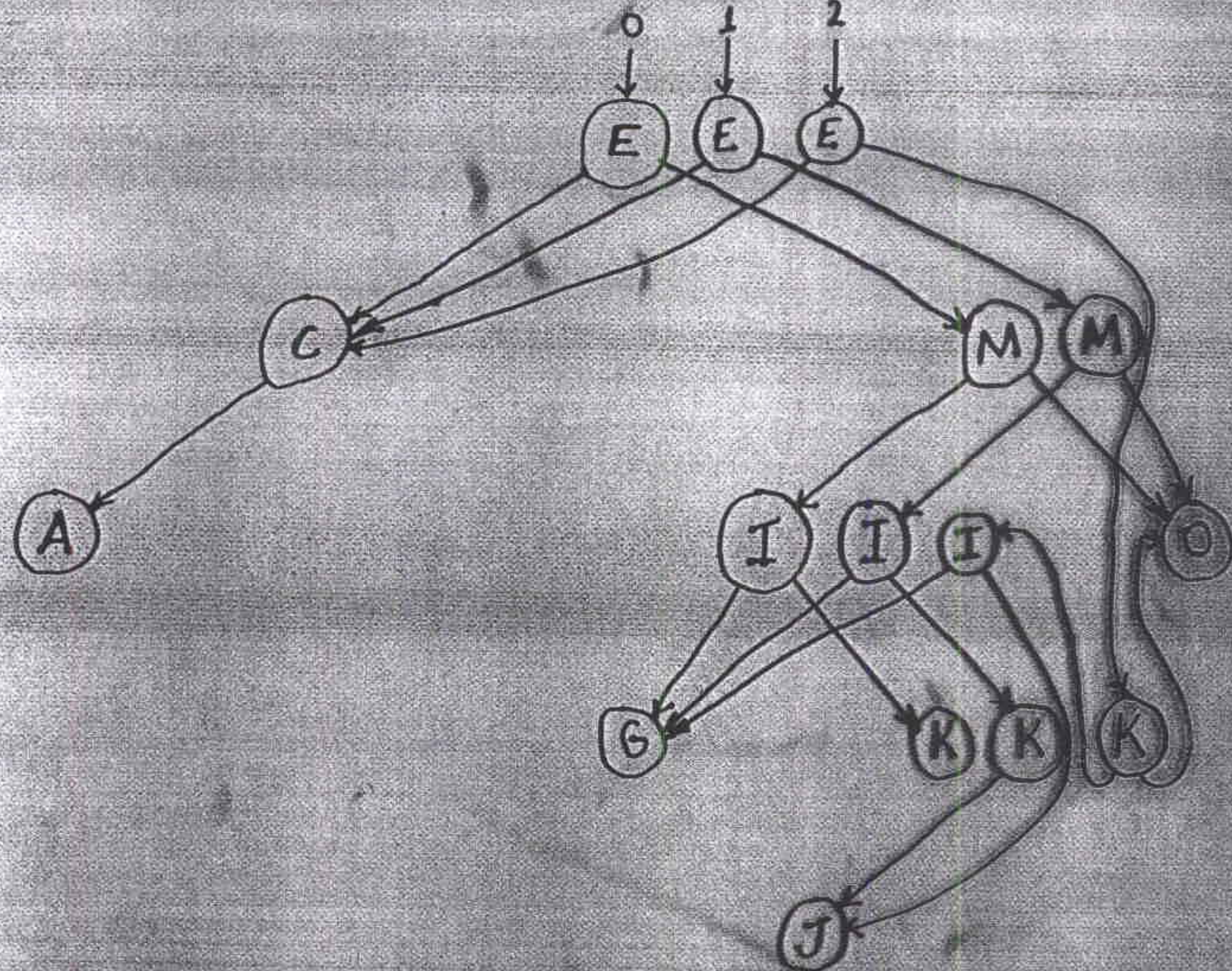
Computational Geometry

Obtaining (Partial) Persistence in Search Trees

Easy Solution: Copy the entire access path and all changed nodes during each update.

Reps, Krijnen and Meertens, Verhofstad

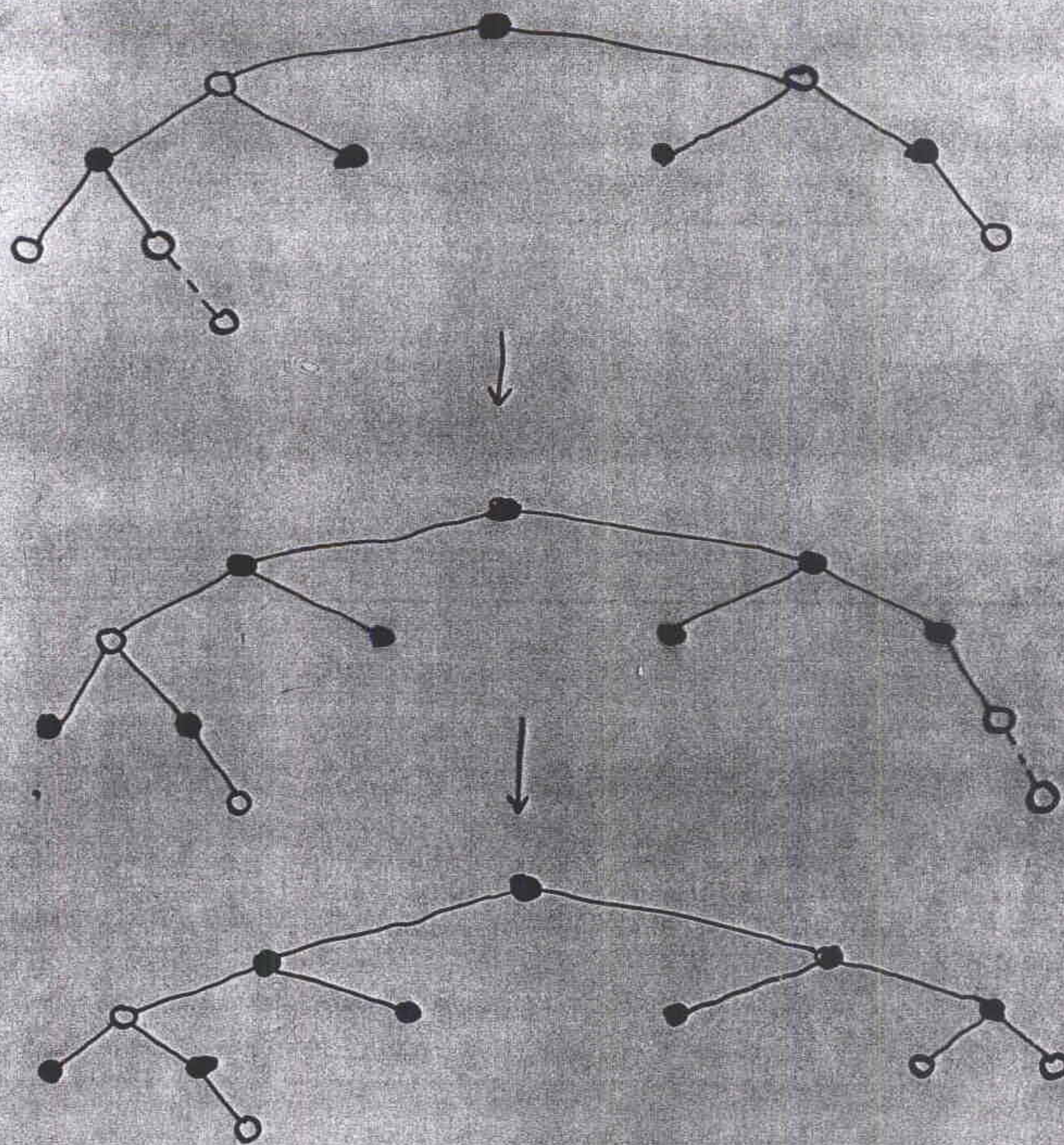
Myers, Swart



initial tree: A, C, E, G, I, K, M, O

iJ, dM

Two Insertions



Ideas in **our** Construction
(partial persistence)

Allow nodes to become arbitrarily "fat."

Each new value stored in a node gets
a field name and a version stamp.

Navigation through the structure
uses binary search on the
version stamps.

Space is $O(1)$ per update step
but
time is logarithmic per access step.

Improving the Time Bound

Allow only a fixed amount of extra space in a node.

When a node becomes full, create a new copy, with newest pointers.

Node copying can proliferate, but amortized time and space is $O(1)$ per update step.

Amortized bound: copying a node consumes a full node; creation of full nodes is $O(1)$ per update step.

Where is amortization used?

To prove space bound of $O(1)$ per update

Needs only an amortized $O(1)$ bound on the structural update time in the ephemeral (non-persistent) data structure

Red-Black Trees give $O(1)$ space bound per insert/delete

Φ = # filled extra slots in live nodes

Computational Geometry

2-D point location

The post office problem: Given n points in the plane, answer queries of the form, given a new point, to which old point is it closest?

An Application to Computational Geometry:

2-D Point Location

The Post Office Problem: Given n points in the plane, construct a data structure such that, for any query point, the closest data point can be found fast.

Planar Point Location

Plane Sweep: dimension reduction –

one space dimension becomes

a time dimension

in data structures –

persistent search trees

Orthogonal Line Intersection